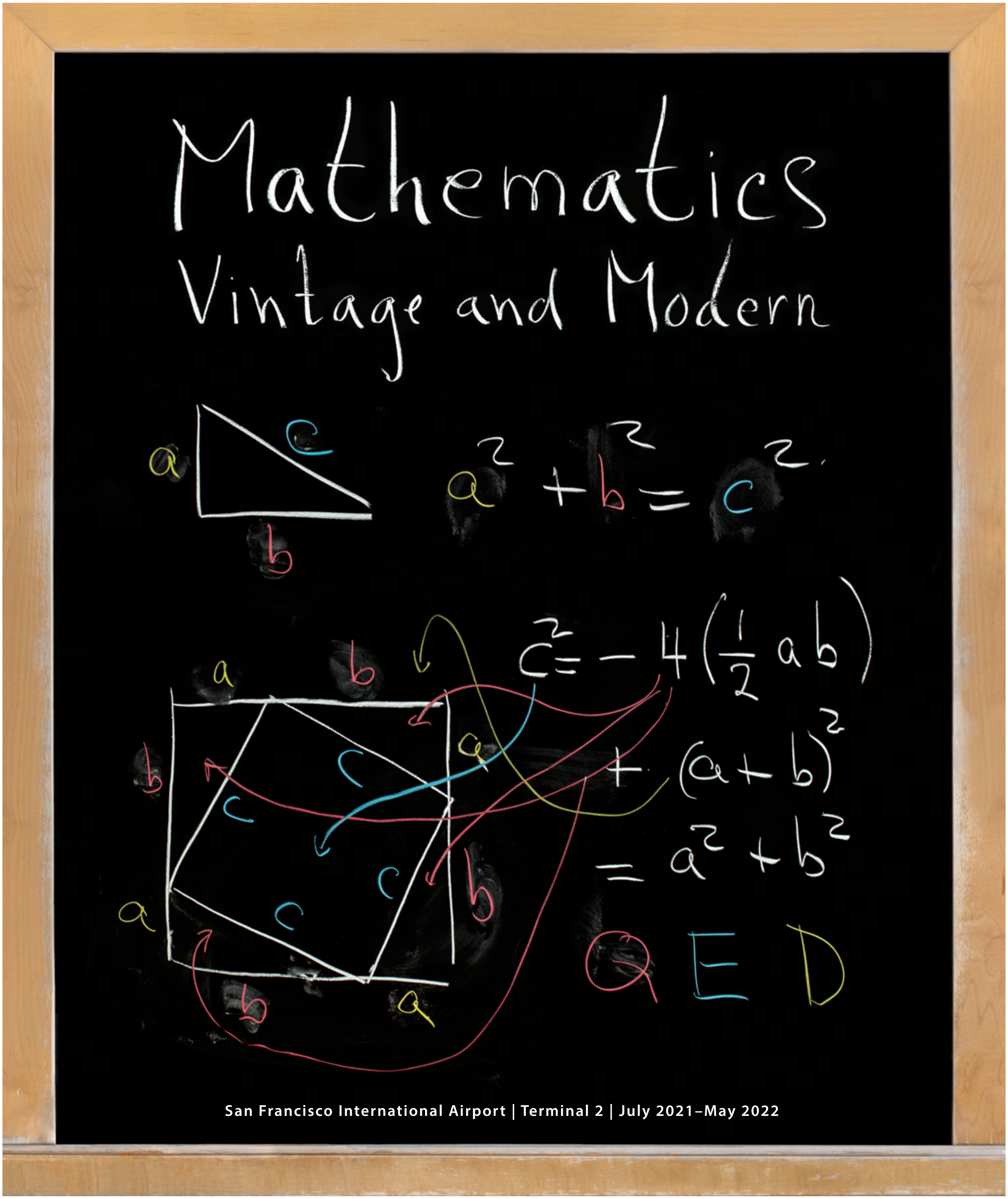
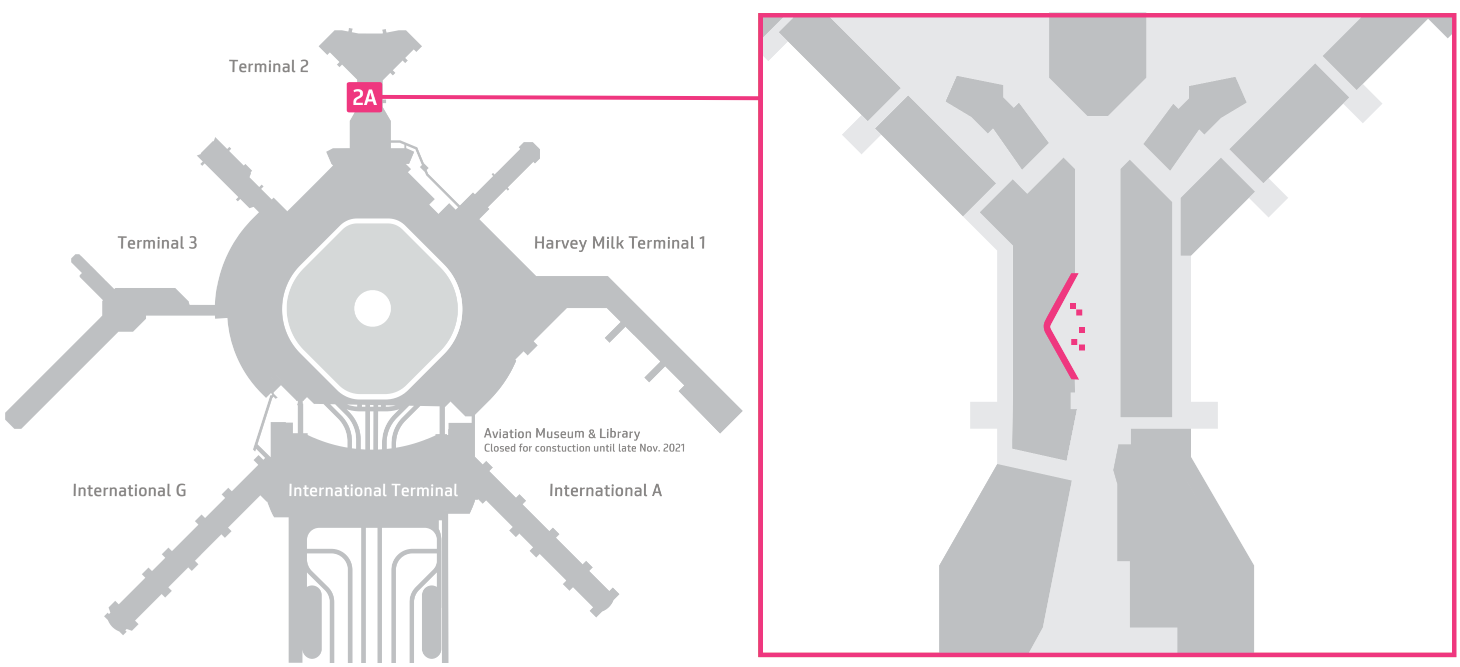
SFO Museum | Education Program



Mathematics
Vintage and Modern

Mathematics: Vintage and Modern Education Program

Thank you for visiting our exhibition in Terminal 2 at the San Francisco International Airport. This education program engages K-12 teachers, students, and the general public with some highlights from *Mathematics: Vintage and Modern*. It includes abbreviated versions of the two large text panels that appear on either end of the gallery as well as focusing on nine themes found throughout the exhibition. This PDF can also be enjoyed at home without viewing the exhibition.



Math is Everywhere

Like music, mathematics is a universal language—understood and used in every culture, civilization, and school. And just as music is much more than notes, math is far more than numbers. Math gives form to logic, reasoning, and intuition. Science and technology grow from mathematics. Pure math includes number theory, algebra, geometry, game theory, topology, analysis, and more. Applied math helps solve real-world problems and provides a foundation for computing, statistics, physics, biology, economics, and engineering. Programs relying on mathematical induction, graph theory, and Boolean algebra lie hidden within your cellphone. Deep math turns up in chess, Rubik's cubes, and even card games.

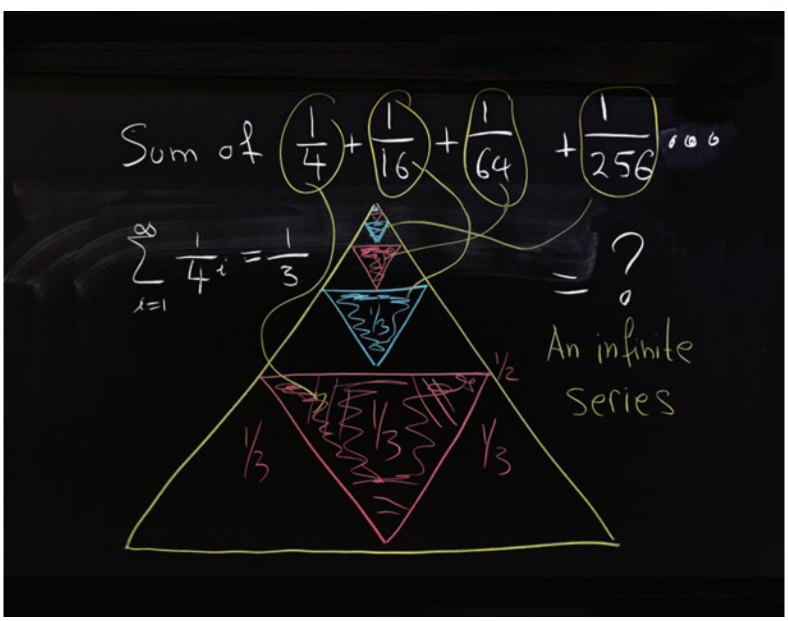
Mathematics permeates nature—symmetries in snowflakes, hexagons in honeycombs, fractals in fern leaves, and concentric circles in tree rings. The Fibonacci sequence, in which each number is the sum of the two preceding ones, occurs in patterns of pinecones and flowers. Faraway planets follow elliptical orbits, and many galaxies form spirals.

The work of mathematicians goes far beyond solving problems. It includes figuring out new ways to express relationships between numbers, simplifying complex problems, understanding connections in datasets, or finding new ways to describe shapes. Mathematics research has reached unprecedented levels. Today your web browser relies on prime numbers for security and cosmologists use topology to understand the shape of our universe. Simultaneously, Silicon Valley tech firms and startups depend on the math community for theoretical and practical support.

Mathematics: Vintage and Modern features objects from the past, when calculation and computation required mechanisms and tables. It also highlights teaching tools that help students learn arithmetic, geometry, and calculus, as well as vintage children's toys and games. Several works of art on exhibit demonstrate complex mathematics through sculptural forms. Examples from modern math, such as knot theory, illustrate how math deals with the very dimensions of space.

This exhibition only touches the surface of mathematics. Every equation tells a story; every answer leads to another question. Many problems puzzle super-computers; others can be solved on your fingers.

Special thank you to co-curator Cliff Stoll; to David Eisenbud, Director of the Mathematical Sciences Research Institute, Berkeley; and to all the lending partners who made this exhibition possible.



Sum of an Infinite Series 2021
(proof without words)
Written by David Eisenbud (b. 1947)
Director of the Mathematical Sciences Research Institute
and Professor of Mathematics,
University of California, Berkeley
R2021.0914.004

Polyhedra

The Greek term combines 'poly' meaning many and 'hedra' for faces or sides; hence, polyhedra are three-dimensional shapes made of polygon faces joined at the edges and corners. Artist Stacy Speyer created these three polyhedra—a hexahedron, a dodecahedron, and a rhombic triacontahedron.

Try this: Choose any polyhedron. Count the number of corners, then subtract the number of edges, and add the number of faces. For convex polyhedra, you will always get two.

For instance, this hexahedron (cube) has eight corners, twelve edges, and six faces: 8 - 12 + 6 = 2. Discovered by mathematician Leonhard Euler in 1758, this equation equals two for all convex polyhedra. This is known as the Euler characteristic, χ, which equals the number of corners, minus the number of edges, plus the number of faces. It demonstrates a topological invariant—a property that will not change as a figure grows, shrinks, twists, or stretches.

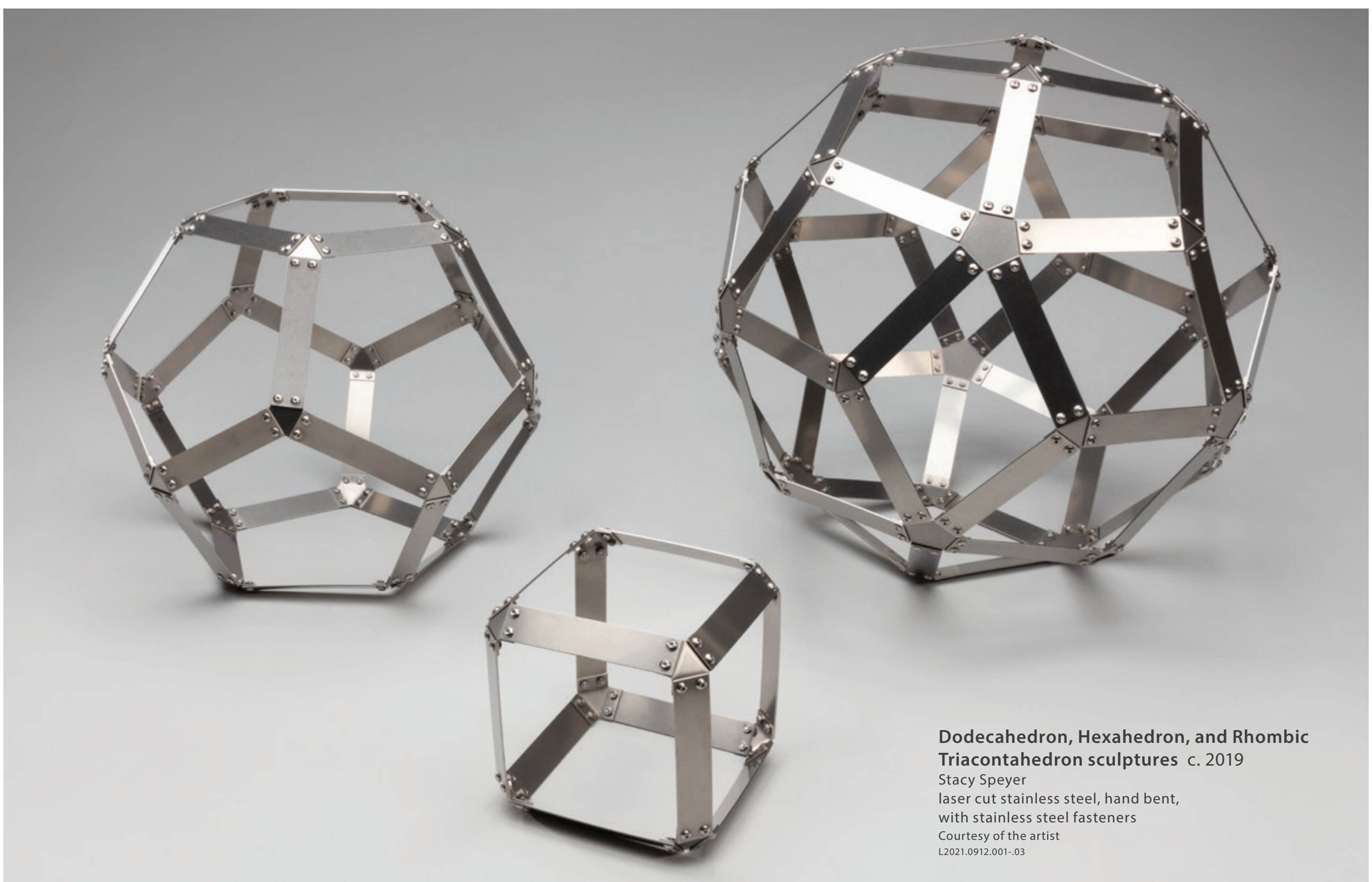
Discussion

What is a polyhedron?

A three-dimensional shape with flat faces, such as a cube

What is a topological invariant?

A property that will not change as a figure grows, shrinks, twists, or stretches



Dodecahedron, Hexahedron, and Rhombic Triacontahedron sculptures c. 2019
Stacy Speyer
laser cut stainless steel, hand bent, with stainless steel fasteners
Courtesy of the artist
L2021.0912.001-.03

Klein Bottles

A mathematician named Klein
Thought the Möbius loop was divine.
Said he: "If you glue
The edges of two,
You'll get a weird bottle like mine."

Mathematician Leo Moser (1921–70)

Intrigued by topology—the study of geometric properties that do not change when you stretch a shape—Cliff Stoll began making Klein bottles. Give a strip of paper a half-twist and tape the ends together— you get a Möbius loop, a one-sided shape with one edge. Make two Möbius loops and glue their edges together—you should get a Klein bottle. But this is impossible in our universe—you need four-spatial dimensions! In our 3D universe, every Klein bottle must have a self-intersection, where tubing crosses through a wall.

This large, classical Klein bottle shows the shape first imagined by German mathematician Felix Klein (1849–1925) in 1882. An ordinary glass bottle has a lip that separates the outside surface from the inside surface. An ant walking on the bottle must cross an edge (the lip) to reach the other side. But a Klein bottle has only one side—an ant can walk the entire surface and never cross an edge.

Discussion

What is topology?

Topology is the study of geometric properties that do not change when you stretch a shape.

Can a Klein bottle be filled with water?

A true four-dimensional Klein bottle cannot be filled; it would enclose no volume. However, the three-dimensional glass Klein bottle can be filled with water.

Tall Klein bottle c. 2005–19
Cliff Stoll (b. 1950)
Glass work by 邢玮 (Ms. Xing Wei) of Shanghai, China
borosilicate glass
Courtesy of the artist
L2021.0902.002

Mathematical Models

From the 1870s to around World War I, universities in Europe and North America acquired collections of mathematical models made from plaster, metal, paper, wood, and string. These models allowed students to see the geometry of complex functions, revolutionary to mathematics at the time. German mathematicians Felix Klein (1849–1925) and Alexander Brill (1842–1935) created some of the earliest models. Klein displayed a number of them at the 1893 World's Columbian Exhibition in Chicago to promote the idea that students must visualize algebraic relationships through geometry. Today, algebraic geometry serves as a source of deep insight throughout mathematics.

Discussion

What is algebraic geometry?

Algebraic geometry once connected simple visual geometry to the equations of algebra. Today, it is expanded to the study of solutions of polynomial equations—a fundamental field of mathematics.

When were geometric models popularly used in universities?

From the 1870s to around 1915

Real part of the Weierstrass elliptic p-function and third order surface (Cayley's Ruled Surface) c. 1880s–90s
L. Brill
Germany
plaster
Courtesy of the Department of Mathematics, University of California, Berkeley
L2021.0913.002-.03

Knot Theory

Long associated with sailors, nautical knots have ends to tie or untie, but in mathematical knots, both ends connect. Some of the simplest mathematical knots include the unknot, which forms a ring, and the trefoil or torus knot, which has three crossings. Knot theory is a branch of mathematical topology—understanding how objects can be deformed by stretching and bending, all the while keeping the same connectivity and structure.

In a single dimension, a string appears straight—it cannot be bent or curved. In two-dimensions, on a tabletop, you can form a curved string, but you cannot tie a knot. In our universe of three-spatial dimensions, you can tie a knot in a string. Curiously, in a universe with four-spatial dimensions, a string passes right through itself—so a knot immediately unties. You could not tie your shoes in a four-dimensional universe!

Discussion

How do mathematicians think about knots?

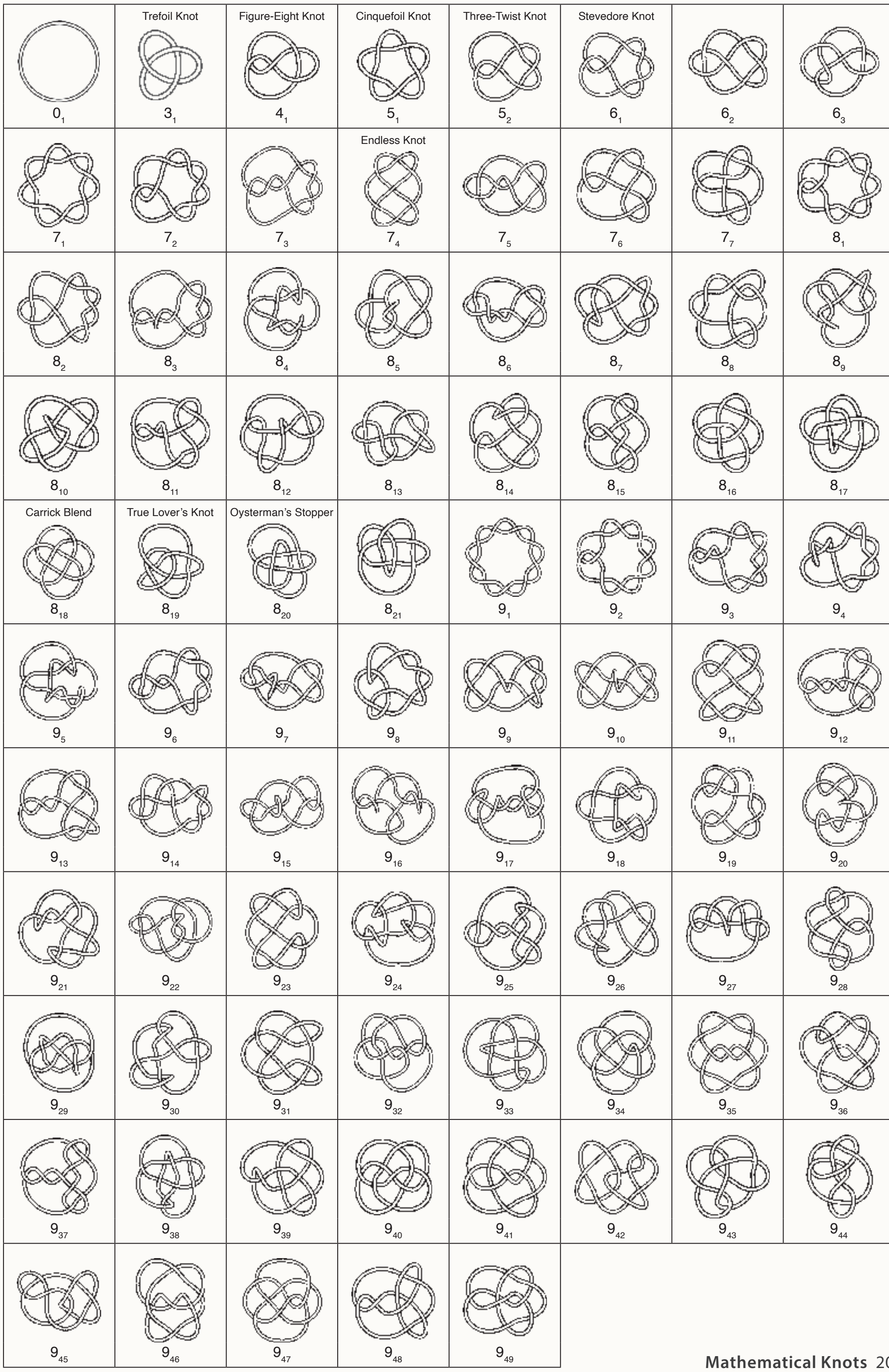
By considering the space around the knot

How can you tell if two knots are actually the same?

This is a surprisingly difficult thing to do! It is one of many tough questions in math.

Torus Knot_3_5 sculpture 2010
Carlo Séquin (b. 1941)
bronze
Courtesy of the artist
L2021.0903.001

Mathematical Knots



Mathematical Knots 2021
Based on The Rolfsen Knot Table
R2021.0926.001

Puzzle Cubes and Group Theory

In 1974, Hungarian sculptor Ernő Rubik (b. 1944) invented the Rubik's cube. The collection of ways to turn a Rubik's cube forms a mathematical group—a structure first used to study symmetries. Group theory, integral to Rubik's cubes, is a powerful tool in mathematics, and is also important in cryptography, chemistry, and robotics.

Group theory shows that a simple Rubik's cube has 43,252,003,274,489,856,000 different arrangements—even more if you consider orientations of each cubie. For many algebraic structures, order of operations is insignificant. For instance, 2+3+4 is the same as 4+2+3, so ordinary addition is commutative. But a Rubik's cube is non-commutative, so order of operations is important: twist the right face clockwise and then the top face clockwise. You cannot undo these two moves by twisting the right face counterclockwise and then the top face counter-clockwise.

"Cubers" memorize algorithms and keep track of each cubie as they solve a scramble. Insight, memorization, dexterity, and lots of practice allow the best speed cubers to solve a cube in under ten seconds.

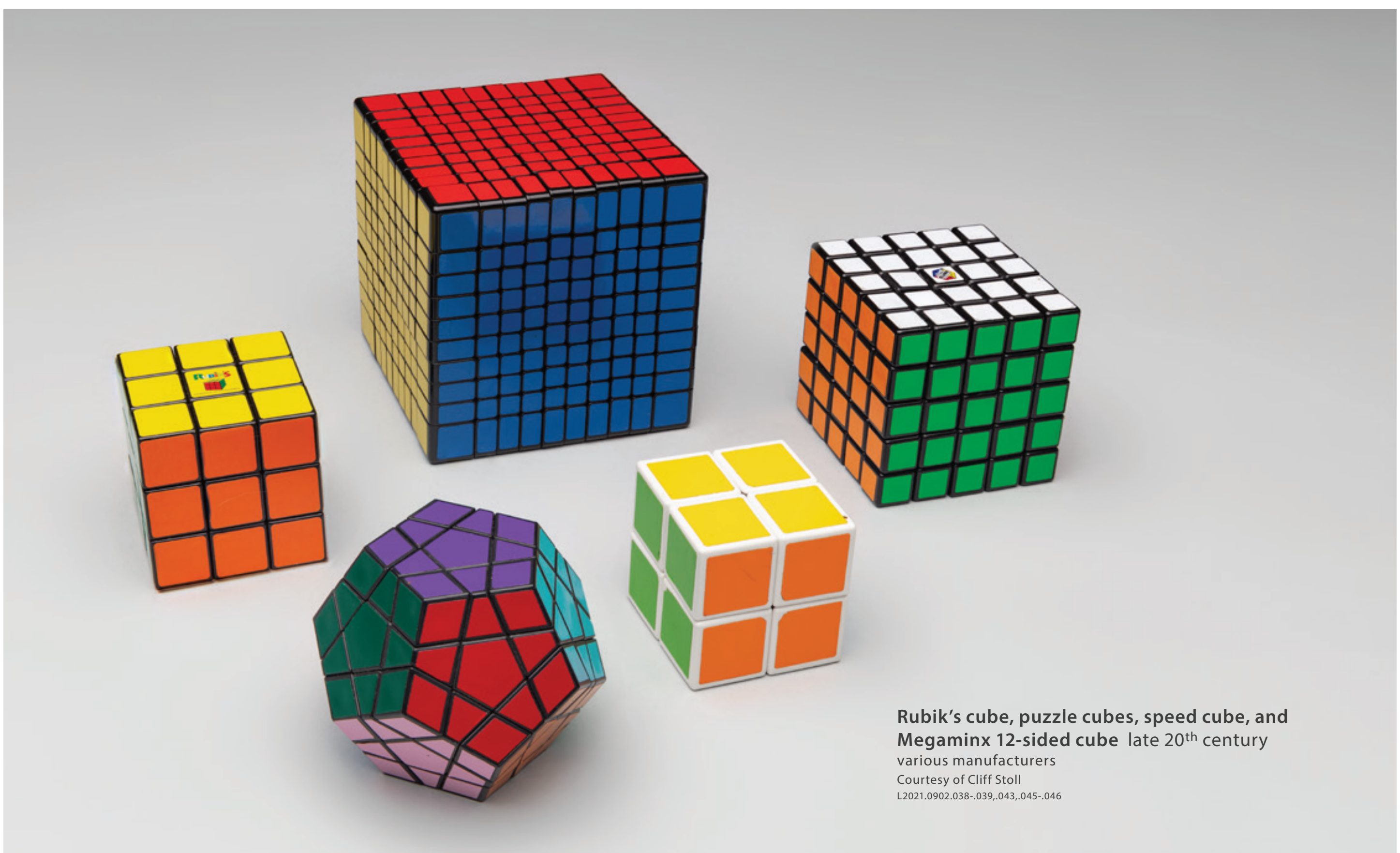
Discussion

What type of math appears in Rubik's cubes?

Group theory is integral to Rubik's cubes—the collection of ways to turn a Rubik's cube forms a mathematical group.

What is an algorithm?

An algorithm is a process or set of rules to be followed in calculations or other problem-solving operations.



Rubik's cube, puzzle cubes, speed cube, and Megaminx 12-sided cube late 20th century
various manufacturers
Courtesy of Cliff Stoll
L2021.0902.038-.039,.043,.045-.046

Computing Before Computers

Throughout the ages, mathematicians have developed ways to calculate—sometimes by finding numerical shortcuts, other times by creating ingenious mechanisms. One of the earliest devices—the abacus—reaches back to antiquity. Used for centuries in East Asia, it consists of rows of moveable beads that represent digits within a frame. Logarithms, invented by Scotsman John Napier (1550–1617) in 1614, substituted addition for multiplication. Englishman William Oughtred (1574–1660), used logarithms to create the first slide rule in 1622. Multiplication and division now required simply lining up two numbers and reading a scale. These analog calculators appeared on the desks and in the pockets of engineers, scientists, and students for most of the twentieth century.

The Arithmometer, patented by Frenchman Charles Xavier Thomas (1785–1870) in 1820, became the first commercially successful mechanical calculator. In 1893, the Millionaire calculator surpassed its predecessor—this innovative machine could directly multiply with a single turn of the hand crank. A few decades later, companies such as Friden and Marchant produced mechanical desktop calculators that could add, subtract, multiply, and divide. Resembling a small pepper grinder, the world's premier pocket calculator, the Curta, debuted in 1947. These instruments proved superior at arithmetic, but in a 1967 televised contest in Hong Kong, a student demonstrated that he could add and subtract faster with his age-old abacus than a woman using a "modern" mechanical calculator.

Throughout World War II and the Space Race of the 1950s and '60s, mechanical calculators and slide rules aided "human computers" whose calculations contributed to groundbreaking projects. NASA's Langley Research Center employed hundreds of female computers from the mid-1930s to the '70s. During the 1940s, Langley began recruiting African American women, who joined the "West Area Computing" unit, an all-Black group of female mathematicians, in 1943.

During the 1970s, electronics pushed mechanisms aside. The HP-35 pocket-sized scientific calculator, introduced in 1972, sparked the end of an era—the handheld computer revolution had begun.

Katherine Johnson (1918–2020) at work with a Monroematic calculator on her desk at Langley Research Center 1962
Hampton, Virginia
Courtesy of NASA
R2021.0918.005

Slide Rules

Slide rules are built on logarithms—adding the logs of two numbers corresponds to multiplying the numbers. Scales on the slide rule allow you to multiply, divide, find exponentials, squares, cubes, and roots. Yet despite their complexity, you cannot perform simple addition and subtraction with a slide rule.

Once the essential tool of engineers, scientists, and industrial designers, the slide rule now pales in comparison to the calculator on your cellphone. Yet the humble "slipstick" helped design countless modern wonders—the Empire State Building, the Boeing 707 airliner, the Hoover Dam, and the curves of the Golden Gate Bridge.

Albert Einstein (1879–1955) preferred to use a modest Nestler slide rule, as did Soviet spacecraft designer Sergei Korolev (1907–66). The Pickett slide rule accompanied American astronauts onboard five Apollo moon flights. The slide rule even aided in designing the devices that rendered it obsolete, including Robert Ragen's (1928–2012) Friden 130 electronic calculator.

Discussion

How do you use a slide rule?

By moving the slider and cursor, you can add or subtract distances on the scales. Because the scales are based on logarithms, adding these distances corresponds to multiplying (or dividing) numbers.

What are some things that engineers and scientists designed with the help of a slide rule?

The Golden Gate Bridge, the Empire State Building, and the Hoover Dam

Lawrence 10-B Slide Rule with *The Slide Rule and How to Use It* book, Rietz 23R slide rule, and *Simplify Math: Learn to Use the Slide Rule* c. 1940s–60s
Courtesy of Chris Hamann
L2021.0901.004.01a,b-.02, 007, .008

Computer at the Lewis Flight Propulsion Laboratory with a slide rule and Friden mechanical calculator 1951
National Advisory Committee for Aeronautics (NACA)
Brook Park, Ohio
Courtesy of NASA
R2021.0918.007

Early Calculating Devices

During the seventeenth and eighteenth centuries, a handful of inventors built mechanical calculating curiosities. The Arithmometer, patented by Charles Xavier Thomas in 1820 in France, became the first commercially successful mechanical calculator in the second half of the nineteenth century. In early models, a silk ribbon activated the machine; later models had a more user-friendly hand crank. Early calculating machines multiplied by repeated addition. To multiple by tens, hundreds, or larger units, users had to shift the carriage and turn the hand crank.

In 1893, Swiss engineer Otto Steigler invented the Millionaire. Made of brass and weighing sixty-seven pounds, this revolutionary machine could perform a direct multiplication. With a single turn of the hand crank, it multiplied two numbers together and calculated results up to eighteen digits. For decades, banks balanced their books and figured compound interest on Millionaires.

Discussion

How did the earliest calculating machines work?

Early calculating machines multiplied by repeated addition; to multiple by tens, hundreds, or larger units, users had to shift the carriage and turn the hand crank.

What did the Millionaire calculating machine do that was different?

It could perform a direct multiplication. With a single turn of the hand crank, it multiplied two numbers together and calculated results up to eighteen digits.

Thomas Arithmometer c. 1850s
Charles Xavier Thomas (1785–1870)
Paris
Courtesy of the Computer History Museum
XB3.76; L2021.0904.012

Millionaire calculating machine c. 1904
Otto Steigler (1858–1923)
Hans W. Egli
Zurich
Courtesy of Cliff Stoll
L2021.0902.012

Curta: The World's First Pocket Calculator

During the 1930s, the smallest calculator weighed several pounds. Austrian engineer Curt Herzstark (1902–88) conceived of a pocket-sized calculator that could add, subtract, multiply, and divide. The son of a Catholic mother and Jewish father, Herzstark was imprisoned in the Buchenwald concentration camp in Germany during World War II where despite all odds, he continued designing his calculator. After being liberated by U.S. troops in April 1945, Herzstark began manufacturing the Curta—the world's first pocket calculator—in Liechtenstein, Europe. From 1947 to 1972, the Curta calculator sold for around $125 and graced the pockets of engineers, accountants, surveyors, and pilots. It does everything that a modern pocket calculator can do except that it is entirely mechanical—no battery, no keypad, or digital display. To add numbers, users simply turn a crank.

Discussion

Why was the Curta revolutionary?

It was the first pocket-sized mechanical calculator.

Who invented the Curta?

Curt Herzstark, whose father was Jewish, invented the Curta; despite all odds, Herzstark continued to work on his design while imprisoned in the Buchenwald concentration camp in Germany during World War II.

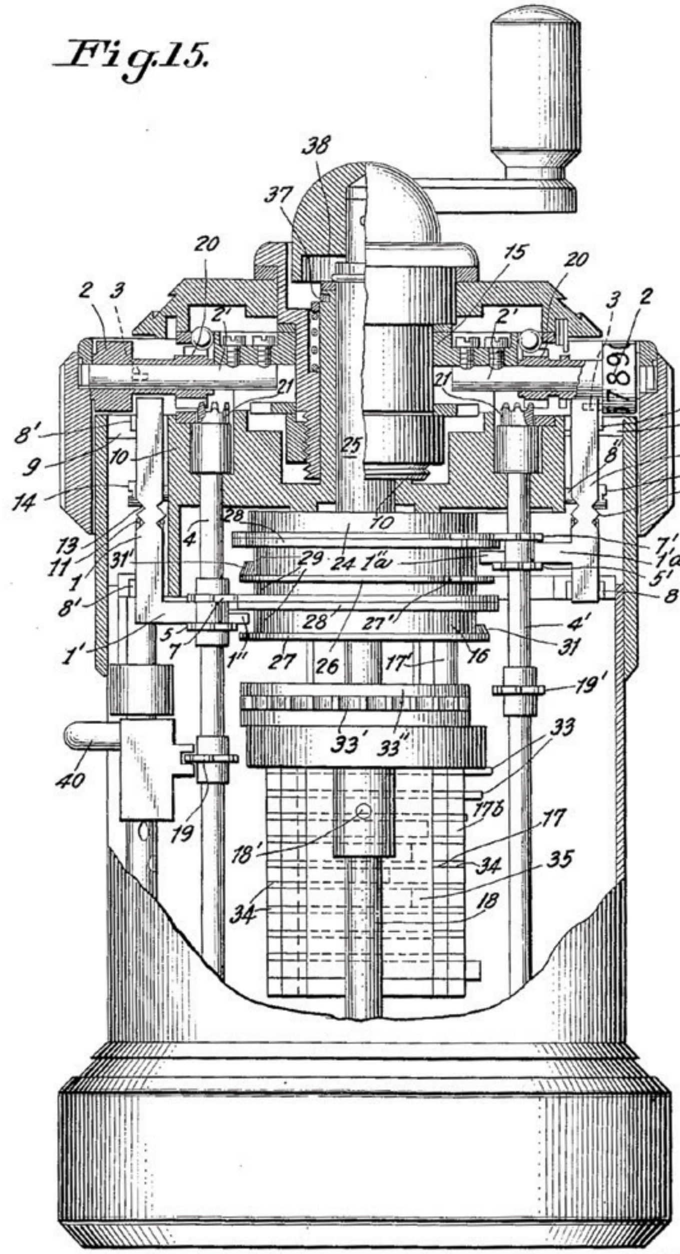
Curta mechanical pocket calculator c. 1950
Contina AG
Liechtenstein
Courtesy of Cliff Stoll
L2021.0902.005

March 11, 1952 C. HERZSTARK **2,588,835**

INDEPENDENT ACTUATOR TENS-TRANSFER MECHANISM

Filed Oct. 27, 1949 4 Sheets-Sheet 4

Fig.15.



INVENTOR
CURT HERZSTARK
BY Singer, Stern & Carlberg
ATTORNEYS

Curta calculator patent drawing 1949
United States Patent and Trademark Office
R2021.0919.001

It’s Electric!

Located in San Leandro, California, the Friden Company manufactured mechanical calculators from 1930 well into the 1960s. In 1963, Robert Ragen (1928–2012), an engineer at Friden, felt that gears and levers were a clumsy way to do simple arithmetic. He designed the Friden 130 to calculate electronically. Among the first electronic calculators, it pioneered an ingenious way to enter data called reverse Polish notation. A user simply typed the first number, pressed enter, then typed the second number and pressed multiply. An immediate success, scientists and engineers felt astounded by its speed: you could multiply two numbers and the result appeared instantly without waiting for motors to start or gears to crunch. Silent and made of lightweight aluminum, the later model 132 even solved square roots.

In 1972, Hewlett-Packard introduced the first successful, scientific pocket calculator—the HP-35—capable of performing trigonometric, logarithmic, and exponential functions, making the slide rule archaic overnight and diminishing the need for desktop calculators.

Discussion

Why was the Friden 130 important?

It was one of the first electronic calculators and pioneered an easy way to enter data called reverse Polish notation.

Why is the HP-35 so significant?

It was the first pocket-sized, scientific, electronic calculator.

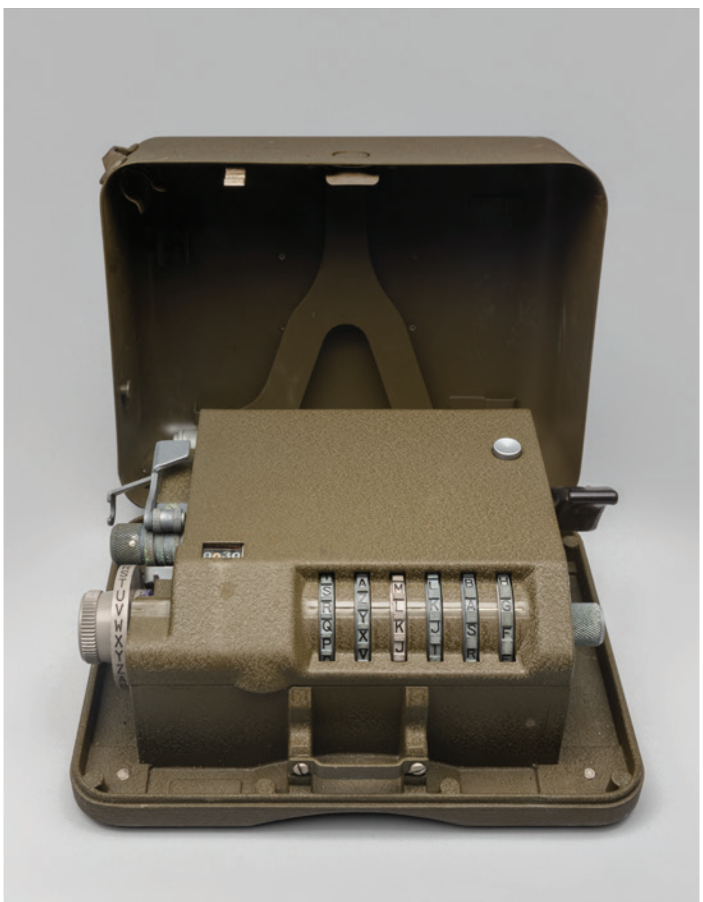
Friden electronic calculator model 132 c. 1965
Friden Calculating Machine Company, Inc.
San Leandro, California
Courtesy of Cliff Stoll
L2021.0902.011

HP-35 pocket calculator 1972
Hewlett-Packard
Palo Alto, California
Courtesy of the Computer History Museum
X771.86; L2021.0904.013

Prime Numbers and Cryptography

2, 3, 5, 7, 11, 13, 17.…Prime numbers, the natural numbers greater than one that are not the product of other natural numbers, have fascinated mathematicians for millennia. Modern cryptosystems rely on huge prime numbers for secure cyphers. Prime numbers prevent outsiders from viewing your data whenever your web browser shows 'https.' Internet security depends on mathematical "trap-door" functions that make it easy to go forward but difficult to reverse. For example, multiplying a pair of big numbers is simple. But finding the primes that factor a huge number is extremely difficult, even for a supercomputer.

During World War II, all sides used mechanical ciphers. The U.S. military's M-209 portable crypto machine required users to encrypt text one letter at a time. By turning a crank, the scrambled message printed out on paper tape. Gears within the M-209 have prime-number ratios to prevent repetitive positions—with over one million positions before the key repeats. The M-209, however, is not fully secure, partly because its prime numbers are short. It worked well for short-lived battlefield messages as the enemy typically deciphered the message too late to be useful.



Discussion

What are prime numbers?

Prime numbers are the natural numbers greater than one that are not the product of other natural numbers: 2, 3, 5, 7, 11, 13, 17.…

How does Internet security use prime numbers for security?

Huge prime numbers are used in security software. It is easy to multiply two big prime numbers, but almost impossible to undo this process (factoring huge numbers into prime factors is a hard problem). These "trap-door" algorithms make it easy for your software to encrypt a secret, but difficult for hackers to decrypt.

M-209 cipher machine c. 1940s
Designed by Boris Hagelin (1892–1983)
Smith & Corona Typewriters, Inc.
Syracuse, New York
Courtesy of the National Cryptologic Museum
L2021.0910.001

A woman operates a foolproof American SIGABA cipher machine, decrypting messages, and intercepting and forwarding enemy coded messages c. 1944
Courtesy of the National Cryptologic Museum
R2021.0910.003

Bibliography

Adams, Colin C. *The Knot Book: An Elementary Introduction to the Mathematical Theory of Knots*. New York: The W. H. Freeman Company, 2001.

Kidwell, Peggy Aldrich, Amy Ackerberg-Hastings, and David Lindsay Roberts. *Tools of American Mathematics Teaching, 1800–2000*. Baltimore: Smithsonian/The John Hopkins University Press, 2008.

Ording, Philip. "Mathematical Objects: The Shapes Behind the Numbers." *Cabinet* Issue 41 (Spring 2011): pp. 51–55.

McLennan, Sarah and Mary Gainer. "When the Computer Wore a Skirt: Langley's Computers, 1935–1970." NASA: https://www.nasa.gov/feature/when-the-computer-wore-a-skirt-langley-s-computers-1935-1970

Stoll, Cliff. "When Slide Rules Ruled." *Scientific American* (May 2006): pp. 80–87.

—— "The Curious History of the First Pocket Calculator." *Scientific American* (January 2004): pp. 82–89.

"Revolution: The First 2000 Years of Computing." Computer History Museum: https://www.computerhistory.org/revolution/

"Science & Mathematics." Smithsonian National Museum of American History: https://americanhistory.si.edu/collections/subjects/science-mathematics

Wilcox, Jennifer. *Sharing the Burden: Women in Cryptography During World War II*. Fort George G. Meade, Maryland: Center for Cryptologic History, National Security Agency, 1998.

Math professor Hilda Geiringer (1893–1973) teaches a class at Wheaton College c. 1953
Norton, Massachusetts
Courtesy of Wheaton College
R2021.0922.001

Mathematics Resources

American Institute of Mathematics
aimath.org

Celebration of Mind
celebrationofmind.org

Creativity in Mathematics
cre8math.com

Julia Robinson Mathematics Festival
jrmf.org

Math Circles
mathcircles.org

Mathematical Sciences Research Institute K-12 Resources
msri.org/web/msri/education/for-k-12-educators/math-circles

Pi Day
piday.org

Stanford University Mathematics Camp
sumac.spcs.stanford.edu

The Bridges Organization
bridgesmathart.org

Young Wonks (Math Circles in California)
youngwonks.com/blog/Math-Circles-in-California

YouTube Channels

Numberphile

Henry Segerman

3Blue1Brown

NancyPi

Stand-up Maths

ViHart